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Bulk viscosity of QCD matter near the critical temperature

Dmitri Kharzeev[a] **and Kirill Tuchin**[bc]

[a] *Department of Physics, Brookhaven National Laboratory, Upton, New York 11973-5000, U.S.A.*
[b] *Department of Physics and Astronomy, Iowa State University, Ames, Iowa, 50011, U.S.A.*
[c] *RIKEN BNL Research Center, Upton, New York 11973-5000, U.S.A.*
E-mail: kharzeev@bnl.gov, tuchin@iastate.edu

Abstract: Kubo's formula relates bulk viscosity to the retarded Green's function of the trace of the energy-momentum tensor. Using low energy theorems of QCD for the latter we derive the formula which relates the bulk viscosity to the energy density and pressure of hot matter. We then employ the available lattice QCD data to extract the bulk viscosity as a function of temperature. We find that close to the deconfinement temperature bulk viscosity becomes large, with viscosity-to-entropy ratio $\zeta/s \sim 1$.

Keywords: Sum Rules, QCD, Lattice QCD, Anomalies in Field and String Theories.

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One of the most striking results coming from RHIC heavy ion program is the observation that hot QCD matter created in $Au - Au$ collisions behaves like an almost ideal liquid rather than a gas of quarks and gluons [1 − 5]. Indeed, hydrodynamical simulations of nuclear collisions at RHIC (see e.g. [6, 7]) indicate that the shear viscosity of QCD plasma is very low even though a quantitative determination is significantly affected by the initial conditions [8]. This observation does not yet have any theoretical explanation due to an enormous complexity of QCD in the regime of strong coupling. This is why the information inferred from the studies of gauge theories treatable at strong coupling such as $N = 4$ SUSY Yang-Mills theory is both timely and valuable. The study of shear viscosity in this theory using the holographic AdS/CFT correspondence has indicated that the shear viscosity η at strong coupling is small, with the viscosity-to-entropy ratio not far from the conjectured bound of $\eta/s = 1/4\pi$ [9, 10] .

However $N = 4$ SUSY Yang-Mills theory is quite different from QCD; in particular it possesses exact conformal invariance whereas the breaking of conformal invariance in QCD is responsible for the salient features of hadronic world including the asymptotic freedom [11], confinement, and deconfinement phase transition at high temperature.[1] Mathematically, conformal invariance implies the conservation of dilatational current s_μ: $\partial^\mu s_\mu = 0$. Since the divergence of dilatational current in field theory is equal to the trace of the energy-momentum tensor $\partial^\mu s_\mu = \theta^\mu_\mu$, in conformally invariant theories $\theta^\mu_\mu = 0$. In QCD, in the chiral limit of massless quarks the trace of the energy-momentum tensor is also equal to zero at the classical level. However quantum effects break conformal invariance [13, 14]:

$$\partial^\mu s_\mu = \theta^\mu_\mu = \sum_q m_q \bar{q} q + \frac{\beta(g)}{2g} \mathrm{Tr}\, G^{\mu\nu} G_{\mu\nu} \,, \tag{1}$$

where $\beta(g)$ is the QCD β-function, which governs the behavior of the running coupling:

$$\mu \frac{dg(\mu)}{d\mu} = \beta(g); \tag{2}$$

note that we have included the coupling g in the definition of the gluon fields and have not written down explicitly the anomalous dimension correction to the quark mass term.

How would this breaking of conformal invariance manifest itself in the transport properties of QCD plasma? How big are the effects arising from it? The transport coefficient of the plasma which is directly related to its conformal properties is the bulk viscosity; indeed, it is related by Kubo's formula to the correlation function of the trace of the energy-momentum tensor:

$$\zeta = \frac{1}{9} \lim_{\omega \to 0} \frac{1}{\omega} \int_0^\infty dt \int d^3 r\, e^{i\omega t} \, \langle [\theta^\mu_\mu(x), \theta^\mu_\mu(0)] \rangle \,. \tag{3}$$

It is clear from (3) that for any conformally invariant theory with $\theta^\mu_\mu \equiv \theta = 0$ the bulk viscosity should vanish.

[1] The effects of conformal symmetry breaking on bulk viscosity of SUSY Yang-Mills theory have been studied in the framework of the AdS/CFT correspondence in ref [12].

The perturbative evaluation of the bulk viscosity ζ of QCD plasma has been performed recently [15], and yielded a very small value, with $\zeta/s \sim 10^{-3}$ at $\alpha_s = 0.3$. The parametric smallness of bulk viscosity can be easily understood from eqs (3) and (1) which show that $\zeta \sim \alpha_s^2$, in accord with the result of ref. [15]. This would seem to suggest that bulk viscosity effects in the quark-gluon plasma are unimportant. However, perturbative expansions at temperatures close to the critical one are not applicable, so at moderate temperatures one has to rely on lattice QCD calculations. Lattice calculations of the equation of state become increasingly precise; however, the direct calculations of transport coefficients have been notoriously difficult. Two calculations have been reported for shear viscosity [16, 17], including a recent high statistics study [17]. Both indicate that η/s is not much higher than the conjectured bound of $1/4\pi$. Fortunately, the correlation function of the trace of the energy-momentum tensor in QCD is constrained by the low-energy theorems, which do not rely on perturbation theory. They can thus be used to express the bulk viscosity in terms of the "interaction measure" $\langle\theta\rangle = \mathcal{E} - 3P$ where $\mathcal{E}$ is the energy density and P is the pressure, which are measured on the lattice with high precision. Such a study is the subject of this Letter.

The calculation of the bulk viscosity starts with the Kubo's formula (3) (we follow the definitions and notations of [18]). Introducing the retarded Green's function we can re-write (3) as

$$\zeta = \frac{1}{9}\lim_{\omega\to 0}\frac{1}{\omega}\int_0^\infty dt \int d^3r\, e^{i\omega t}\, iG^R(x) \;=\; \frac{1}{9}\lim_{\omega\to 0}\frac{1}{\omega}\, iG^R(\omega,\vec{0}) \;=\; -\frac{1}{9}\lim_{\omega\to 0}\frac{1}{\omega}\,\mathrm{Im}G^R(\omega,\vec{0})\,. \tag{4}$$

The last equation follows from the fact that due to P-invariance, function $\mathrm{Im}G^R(\omega,\vec{0})$ is odd in ω while $\mathrm{Re}\,G^R(\omega,\vec{0})$ is even in ω. Let us define the spectral density

$$\rho(\omega,\vec{p}) = -\frac{1}{\pi}\,\mathrm{Im}G^R(\omega,\vec{p})\,. \tag{5}$$

Using the Kramers-Kronig relation the retarded Green's function can be represented as

$$G^R(\omega,\vec{p}) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\mathrm{Im}G^R(u,\vec{p})}{u-\omega-i\varepsilon}\,du = \int_{-\infty}^{\infty}\frac{\rho(u,\vec{p})}{\omega-u+i\varepsilon}\,du \tag{6}$$

The retarded Green's function $G^R(\omega,\vec{p})$ of a bosonic excitation is related to the Euclidean Green's function $G^E(\omega,\vec{p})$ by analytic continuation

$$G^E(\omega,\vec{p}) = -G^R(i\omega,\vec{p})\,, \quad \omega > 0\,. \tag{7}$$

Using (6) and the fact that $\rho(\omega,\vec{p}) = -\rho(-\omega,\vec{p})$ we recover

$$G^E(0,\vec{0}) = 2\int_0^\infty \frac{\rho(u,\vec{0})}{u}\,du\,. \tag{8}$$

As we discussed above, the scale symmetry of QCD lagrangian is broken by quantum vacuum fluctuations. As a result the trace of the energy momentum tensor θ acquires a non-zero vacuum expectation value. The correlation functions constructed out of operators

$\theta(x)$ satisfy a chain of low energy theorems (LET) which are a consequence of the renormalization group invariance of observable quantities [19]. These low-energy theorems entirely determine the dynamics of the effective low-energy theory. This effective theory has an elegant geometrical interpretation [20]; in particular, gluodynamics can be represented as a classical theory formulated on a curved (conformally flat) space-time background [21]. At finite temperature, the breaking of scale invariance by quantum fluctuations results in $\theta = \mathcal{E} - 3P \neq 0$ clearly observed on the lattice for SU(3) gluodynamics [22]; the presence of quarks [23] including the physical case of two light and a strange quark [24], or considering large N_c [25] does not change this conclusion.

The LET of ref. [19, 20] were generalized to the case of finite temperature in [26, 27]. The lowest in the chain of relations reads (at zero baryon chemical potential):

$$G^E(0,\vec{0}) = \int d^4x \, \langle T\theta(x), \theta(0)\rangle^f = \left(T\frac{\partial}{\partial T} - 4\right) \langle\theta\rangle_T^f \, ; \tag{9}$$

the superscript "f " stands for the finite part, as we shall now explain. The derivation of these low-energy theorems is based upon the renormalization group invariance of the physical expectation value of θ which implies that the r.h.s. of (9) must not depend on the renormalization scale M_0. This means that the renormalization scale M_0 can only enter in the RG-invariant combination; at one-loop level, this is $[M_0 \exp(-8\pi^2/bg_0^2)]^d$, where b is the coefficient of the beta-function, g_0 is the bare coupling constant and d is the dimension of space-time (canonical dimension of θ). Perturbation theory at $T = 0$ on the other hand would not obey this requirement; it gives the divergent contribution $\sim \Lambda_{\rm UV}^4$, where $\Lambda_{\rm UV}$ is the ultra-violet cutoff. Therefore, the *finite* expectation value of $\langle\theta\rangle_T^f$ must be defined as the difference between the (infinite) expectation value of $\langle\theta\rangle_T$ and the (infinite) expectation value of $\langle\theta\rangle_{T=0}^{p.t.}$ computed at $T = 0$ in perturbation theory [19, 26, 27]:

$$\langle\theta\rangle_T^f = \langle\theta\rangle_T - \langle\theta\rangle_{T=0}^{p.t.}. \tag{10}$$

To relate the thermal expectation value of $\langle\theta\rangle_T$ to the quantity $(\mathcal{E} - 3P)_{\rm LAT}$ computed on the lattice, we should also keep in mind that

$$(\mathcal{E} - 3P)_{\rm LAT} = \langle\theta\rangle_T - \langle\theta\rangle_0, \tag{11}$$

i.e. the zero-temperature expectation value of the trace of the energy-momentum tensor

$$\langle\theta\rangle_0 = -4|\epsilon_v| \tag{12}$$

has to be subtracted; it is related to the vacuum energy density $\epsilon_v < 0$. Now, using (8), (9) and (11) we can write down the following spectral representation:

$$2\int_0^\infty \frac{\rho^f(\omega,\vec{0})}{\omega} \, d\omega = -\left(4 - T\frac{\partial}{\partial T}\right) \langle\theta\rangle_T^f, \tag{13}$$

where the spectral density $\rho^f(\omega,\vec{0})$ is the difference between the physical, finite temperature, and perturbative, zero temperature, spectral densities of the correlation function of θ:

$$\rho^f(\omega,\vec{0}) = \rho(\omega,\vec{0}) - \rho_{T=0}^{p.t.}(\omega,\vec{0}). \tag{14}$$

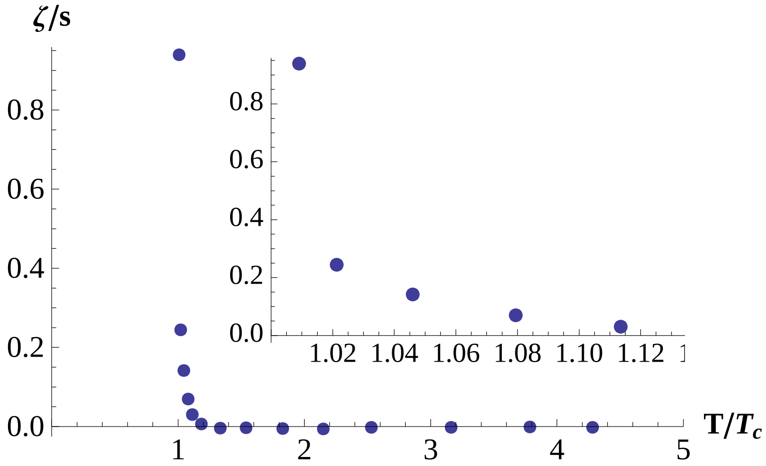

Figure 1: The ratio of bulk viscosity to the entropy density for SU(3) gluodynamics. We have used $|\epsilon_v| = 0.62\, T_c^4$ and $T_c = 0.28\,\mathrm{GeV}$ [22].

At leading order, the perturbative spectral density behaves[2] as $\rho^{p.t.}_{T=0}(\omega,\vec{0}) \sim \alpha_s^2\,\omega^4$. At high frequencies $\omega \gg T$ the physical spectral density should be the same as at zero temperature, and in an asymptotically free theory will become perturbative, so that the difference $\rho^f(\omega,\vec{0})$ will vanish.

Since the thermal expectation value $\langle\theta\rangle_T$ is related to the "interaction measure" $(\mathcal{E}-3P)_{\mathrm{LAT}}$ computed on the lattice, we can derive the following sum rule

$$2\int_0^\infty \frac{\rho^f(\omega,\vec{0})}{\omega}\,du = -\left(4 - T\frac{\partial}{\partial T}\right)\langle\theta\rangle^f_T = T^5\frac{\partial}{\partial T}\frac{(\mathcal{E}-3P)_{\mathrm{LAT}}}{T^4} + 16|\epsilon_v|\,, \tag{15}$$

This *exact* relation is the main result of our paper.

We would like to emphasize again that the integral over the frequency on the l.h.s. of (13) is convergent. This follows from a general theorem of finite-temperature field theory (see e.g. [29], section 3.5) stating that the renormalization at zero temperature suffices to make the theory finite at finite temperatures. Therefore it is sufficient to regularize the correlation function of θ at $T=0$. Indeed, the finite-temperature part of the correlation function falls off as $\exp(-\omega/T)$ at large $\omega \gg T$ [29], so no temperature-dependent term in the spectral density can survive at large frequencies ω. Since the UV divergent part of the $T=0$ correlation function at large frequencies is given by perturbation theory, the subtraction of $\rho^{p.t.}_{T=0}(\omega,\vec{0})$ must make the integral in (15) convergent.

In order to extract the bulk viscosity ζ from (15) we need to make an ansatz for the spectral density ρ^f. In the small frequency region, we will *assume* the following ansatz

$$\frac{\rho^f(\omega,\vec{0})}{\omega} = \frac{9\,\zeta}{\pi}\frac{\omega_0^2}{\omega_0^2+\omega^2}\,, \tag{16}$$

[2]For an explicit perturbative expression and a discussion of the properties of $\rho(\omega,\vec{0})$ at small frequencies see e.g. [28].

which satisfies (4) and (5). Substituting (16) in (13) we arrive at

$$\zeta = \frac{1}{9\,\omega_0}\left\{T^5\frac{\partial}{\partial T}\frac{(\mathcal{E}-3P)_{\rm LAT}}{T^4} + 16|\epsilon_v|\right\}. \qquad (17)$$

A peculiar feature of this result is that the bulk viscosity is linear in the difference $\mathcal{E}-3P$, rather than quadratic as naively implied by the Kubo's formula. This is similar to the strong coupling result obtained for the non-conformal supersymmetric Yang-Mills gauge plasma [12].

As we already discussed, at some frequency $\omega \gg T$ the spectral density $\rho(\omega,\vec{0})$ should approach the spectral density at zero temperature [29], and in an asymptotically free theory will thus be given by the zero-temperature perturbation theory. Because of this the integral over the difference given by (14) will always converge. Let us define the parameter $\omega_0 = \omega_0(T)$ as a scale above which the contribution of the difference given by (14) to the sum rule becomes negligible. On dimensional grounds, we expect it to be proportional to the temperature, $\omega_0 \sim T$.

In our calculation, we estimate it as the scale at which the lattice calculations of the running coupling [30] coincide with the perturbative expression. In the region $1 < T/T_c < 3$ we find $\omega \approx (T/T_c)$ 1.4 GeV. Now we are ready to use (17) to extract the bulk viscosity from the lattice data.

The results of the numerical calculation using as an input the high precision lattice data [22] are displayed in figure 1. One can see that away from T_c the bulk viscosity is small, in accord with the expectations based on the perturbative results [15]. However, close to T_c the rapid growth of $\mathcal{E}-3P$ causes a dramatic increase of bulk viscosity. Basing on the lattice results [16, 17] which indicate that the shear viscosity remains small close to T_c, we expect that bulk viscosity will be the dominant correction to the ideal hydrodynamical behavior in the vicinity of the deconfinement phase transition.

Let us now discuss the uncertainties associated with our method. Since the basic relation (13) is exact, all of the uncertainties are associated with the ansatz (16) for the spectral density. Admittedly, we do not have a good theoretical control over the form of the spectral density, and this results in some numerical uncertainty in the value of the extracted bulk viscosity. Nevertheless, the rapid increase of ζ/s close to T_c is a reflection of the sharp growth of $(\mathcal{E}-3P)_{\rm LAT}/T^4$ present in the lattice data and is a general feature of our result. Moreover, lattice studies of the spectral densities in the vector channel indicate that the ansatz (16) is a reasonable description of the low-frequency behavior [31].

Since bulk viscosity describes the response of a system to expansion, its rapid increase near the QCD phase transition would slow down the expansion of hot matter produced in heavy ion collisions and terminate the "perfect liquid" hydrodynamical behavior of the plasma at the hadronization point, producing extra entropy. Likewise, the Early Universe would perhaps slow down its expansion and produce additional entropy at the QCD phase transition due to the presence of large bulk viscosity. A rapid increase in the correlation function of the trace of the energy-momentum tensor most likely indicates the presence of a light scalar color-singlet gluonic excitation in QCD matter close to the phase transition. We

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leave a detailed study of the dynamics of this mode and its phenomenological implications for the future.

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Note added: after the submission of our manuscript to the preprint database, a number of important developments have taken place, and we would like to briefly review them. The lattice study of bulk viscosity has been performed by Meyer [32], indicating that the bulk viscosity of quark-gluon plasma indeed rapidly increases near the deconfinement temperature. The sum rule analysis presented here has been extended to QCD with almost physical quark masses [36], using the recent lattice results on the equation of state [37]. Bulk viscosity has also excited considerable attention in the context of gauge theory-gravity duality; a computation of bulk viscosity requires a deformation of the conformal metric of AdS_5 space, and has been performed by Gubser et al [33, 34]. The authors also observe an increase of bulk viscosity near the deconfinement temperature [33, 34], with the numerical values somewhat smaller than reported in this paper. Buchel [35] has proposed a lower bound on bulk viscosity in the context of a holographic approach.

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